Navigation of the Shackleton Expedition on the Weddell Sea pack ice

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The Imperial Trans-Antarctic Expedition under the leadership of Sir Ernest Shackleton departed South Georgia bound for the Antarctic on 5 December 1914. On 19 January 1915, their ship, *Endurance*, became caught in the pack ice of the Weddell Sea and drifted with it until being crushed, sinking on 21 November. While the Expedition remained in the grip of the ice, observations continued to be made for navigational purposes. Being out of sight of land for a long period meant that there was no easy way to rate the chronometers and their drift caused the estimated longitude to become increasingly uncertain. On 24 June a series of observations of lunar occultations of fixed stars was begun as an absolute way of determining Greenwich Mean Time. These, along with other observations, are recorded in the Expedition's original logbooks that are housed at Canterbury Museum in Christchurch, New Zealand. The logs have been examined and the navigational methods used while on the sea ice are described in detail here. The calculations of a few key pages have been replicated and annotated to act as a glossary to facilitate reading the other entries.

Keywords: celestial navigation, chronometer, dead reckoning, Ernest Shackleton, ex-meridian sight, Frank Worsley, Imperial Trans-Antarctic Expedition, lunar occultation, Reginald James, time sight, Weddell Sea.

Introduction

On 8 August 1914, the Imperial Trans-Antarctic Expedition under the leadership of Sir Ernest Shackleton set sail aboard their vessel *Endurance* from Plymouth, England with the goal of traversing the Antarctic continent from the Weddell to Ross Seas. *Endurance* was under the command of New Zealand-born Captain Frank A Worsley who in subsequent events would prove to be a superlative navigator even under the most adverse of conditions. After making stops at Buenos Aires, Argentina on 26 October and Grytviken whaling station, South Georgia on 5 November, the Expedition encountered pack ice in the Weddell Sea on 7 December and became trapped on 19 January 1915. The *Endurance* began to sustain damage from the pressure of the ice and was abandoned on 27 October before being eventually crushed, sinking on 21 November. The Expedition set up camp on the sea ice, which carried them north until it began to break up within sight of Clarence and Elephant Islands. After a 7 day passage, their three small boats landed on the latter on 15 April 1916.

Navigational records from the Expedition are contained in Worsley's logbooks and loose leaf pages in Canterbury Museum's collection. These have been examined and the navigational methods employed while trapped in, and camped on, the ice are described in detail here.

As the polar night descended, the standard noon sight of the Sun for latitude and time sight for longitude were replaced by meridian and ex-meridian sights for latitude and time sights of stars. The relative stability of the ice meant that observations could be performed by theodolite rather than by sextant. Reginald James, expedition physicist, wrote 'For astronomical

Figure 1. Captain Frank Worsley (left) and Reginald James taking sights from the sea ice by the stern of *Endurance*. Worsley cradles a chronometer or stopwatch and James uses the theodolite to measure the altitude of a star. Scott Polar Research Institute, University of Cambridge, with permission. Photograph by Frank Hurley, 1915. All rights reserved.

observations the sextant or a theodolite was used. The theodolite employed was a light 3" Vernier instrument by Carey Porter intended for sledging work' (Shackleton 1920 Appendix I). James and Worsley are shown making observations in Figure 1.

Accurate time keeping by the careful calibration or rating and maintenance of the ship's chronometers was crucial for finding longitude. Being out of sight of known landmarks for long periods meant that the uncertainty in the chronometer error (CE) grew over time and with it the uncertainty in longitude. In the early twentieth century, prior to the general availability of radio time signals, an uncertainty in the CE anywhere from ½ to 2 seconds per day since the last rating could be expected. From 24 June to 15 September 1915, a series of timings of lunar occultations of fixed stars were undertaken from which the chronometer errors and rates could be reliably determined. The appearance of Mount Percy on Joinville Island in late March 1916 presented new possibilities to confirm the chronometer errors.

In the section entitled 'The Logbook' a few of the key recorded events have been extracted. 'Navigation from *Endurance* in the Pack Ice' explains how ex-meridian and time sights of stars were used during the Antarctic winter to determine position. The section 'Longitude by Occultation of a Fixed Star' examines the timings that were made and traces the background of the methods used to reduce them. In this and the previous sections log entries on some selected dates are closely examined and replicated. 'Navigation at Ocean and Patience Camps' principally describes the efforts at position finding using Mount Percy on Joinville Island. 'The Chronometers' collects what information is available concerning the ship's chronometers and their rating process. The log entries pertaining to the passage from the icepack to Elephant Island have been transcribed and replicated in Appendix A.

A transcription and replication of the log entries that relate to the famous voyage of the *James Caird* from Elephant Island to South Georgia can be found in Bergman et al. (2018).

While the Shackleton Expedition constitutes an epic tale of Antarctic survival, the present work focuses primarily on the log book entries and technical details of the navigational methods that were used. Comprehensive accounts covering the events of the Expedition as a whole can be found elsewhere (Shackleton 1920; Worsley 1998; Alexander 2001; Lansing 2014).

The Logbook

Logbook entries follow the common standard of recording noon sights for latitude and time sights for longitude which were advanced or retarded to local noon by Dead Reckoning (DR) to give the ship's observed position at noon (Bergman et al. 2018). Weather conditions, sounding data and noteworthy events are also recorded. The log entries for 21 June, 26 July (Worsley 1915: 129, 166) and 3 October 1915 (Worsley 1916: 30) appear to show attempts to obtain lines of position using what then was known as the 'new navigation'. The graphical solutions are found on the following pages.

Entrapment in the Ice and the loss of *Endurance*

The log entry for 19 January 1915 (Worsley 1915: 64) contains the ominous words "Fast in pack", which is repeated daily until 2 February, when it becomes "F. in P." (Worsley 1915: 66) and is eventually omitted completely.

In late October 1915 (Worsley 1916: 36, 38), the following events are recorded and included an illustration of the *Endurance* trapped in ice (Fig. 2) on the entry dated 24 October 1915 and a table that recorded course and distance to possible destinations (Table 1):

24th [October, 1915] Strong SSE to SE breeze Cloudy & Misty
6.45PM Ship sustained heavy pressure thru having got pushed into a bad angle of floes & pressure ridges which then moved in direction of arrows twisting sternpost

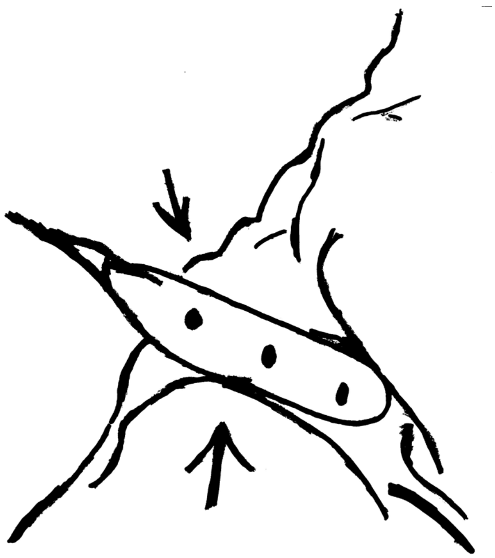

Figure 2. Copy of Worsley's sketch of *Endurance's* position in the ice pack from the log entry of 24 October, 1915.

against floe on star. quarter, starting hidden ends of planking & making ship leak badly. Rigged main pumps, got up steam & started bilge pump at 8PM. All hands watch & watch pumping ship & assistg Carp make coffer dam all night

Monday 25th
Strong SE breeze Cloudy & Misty Carp working day & night on coffer dam

Tuesday 26th Oct
7PM Very heavy pressure with twistg strain, rackg ship & openg butts of planks 3" & 4" stard side 9P Lowered boats gear sledges & provisions on floe. Midnight working on floes closed leak slightly. All hands pumping all night
Mod to gentle S/W to SSW breeze Blue sky and clouds
Temperture 0 –to –15°

Wednesday 27th Oct
Gentle SSE to SSW breeze
4P Terrific pressure heaving stern up 9 feet smashing rudder, rudder post & stern post. Decks breaking up 7P. Ship too dangerous to live in we are forced to abandon her. Water overmastering pumps & coming up to fires draw fires & let down steam
Men & dogs sleep on floe.
S Lat W Long
Landed on Floe 69°5' 51° 32' 27. X.15

Later in the log book, Worsley (1916: 45) wrote:

Sunday Nov.r 21st
1915 68°39'30" 52°26'30"
Endurance sank after drifting in 306 days
N38°E 605 miles
Total dists added together between Obs Posns
= 1250 miles

Endurance				69 °	5 ′ S	51 °	32 ′ W
Paulet	N 17° W	346	miles	63 °	35 ′ S	55	52 ′ W
Snow H	N 25° W	312	"	64	22	57	6
Weather I	N 55° W	272	"	66	29	61	21
C Dundas Laurie I.S.O.	N 20° E	534	"	60	43	44	22
Estimd W to Barrier	West	182	"	69	5	60	2
Thence to Snow Hill	N 14° E	292	"				

Table 1. From the log entry of 27 October, 1915: course and distance to possible destinations from the location of *Endurance* when she was abandoned. Latitudes and longitudes as known to Worsley appear to the right.

Navigation from *Endurance* in the Pack Ice

When *Endurance* became trapped "fast in pack", navigational practice continued largely as before with noon sights and time sights of the Sun being taken for position. As the polar winter approached the Sun sank lower in the sky until its true altitude could no longer be reliably determined. A last Sun sight was recorded on 11 April 1915, but from then until 28 August, except for a Jupiter sight on 19 April, celestial navigation was entirely conducted using stars. This requires the calculation of sidereal time but their use is otherwise closely analogous to noon sights for latitude and time sights of the Sun for longitude. Circumpolar stars introduce the possibility of observing lower meridian transits below the pole. Catching a star exactly at meridian passage can be challenging. Lecky (1918: Part II, Chapter VIII) advises that "to watch for the transit of a star on a dark night requires no little patience, and it has a decidedly fatiguing effect on the eye". He points out that "From the slowness of their motion in altitude, it follows that the stars near either pole are the best adapted for this observation". These are known as 'ex-meridian sights'. The altitude of a celestial body is observed near to transit and, knowing the declination, hour angle and an approximate latitude, a correction is calculated to determine its altitude at meridian passage. Since the correction is small a high degree of accuracy is not required in computing it.

A final round of sights Rigel - λ Scorpii for longitude were made on 15 October 1915, but from then on they were again made exclusively by the Sun.

As noted previously, many of the sights were taken with the theodolite that, unlike the sextant, is free from the requirement of reliably identifying the horizon. Sights made at very low temperatures, however, present special difficulties. James (Shackleton 1920: Appendix 1) wrote:

> *The chief uncertainty in this measurement is that introduced by the refraction of light by air. At very low temperatures, the correction to be applied on this account is uncertain, and, if possible, observations should always be made in pairs with a north star and a south star for latitude, and an east star and a west star for longitude. The refraction error will then usually mean out.*

On 15 September, the log (Worsley 1916: 17) contains the note:

> *all sextant & theodolite observations … refraction has been corrected by Table IV "Hints to Travellers" [Corrections of the Mean Refraction for the Height of the Thermometer]. This correction proving in [pract]ice far too small has been multiplied by 3 & inter.…*

Figure 3 shows the log entry for 4 July 1915, which is fairly typical for the period. It records the temperature as being -8° Fahrenheit (-22°C) and position obtained from the recorded observations as being "74°9'S 48°57'W". The double underline is used throughout the log to denote a final observed position and distinguishes it from intermediate calculations. A single underline denotes a position found by Dead Reckoning. The drift of S 71° W 28 nautical miles (52 km) is computed for 7 days since the last fully observed position was obtained on 27 June. On the intervening days, the position is estimated by single latitude sights on 28 June and 3 July and otherwise by Dead Reckoning. Sounding data is recorded as "203fms gl.m." indicating depth of 203 fathoms (371 m) and a bottom composed of glacial mud[1].

The direction of drift was estimated (Shackleton 1920: Appendix 1) using a device constructed by Worsley that consisted of a vane at the end of an iron rod passing through a tube in the ice into the sea below. The speed of the drift could be estimated by noting how quickly the vane returned to its original position when displaced. The speed and direction of drift could also be estimated by noting the trend of the wire when a sounding was taken. The log contains a diagram (Worsley 1915: 77) in which this is carried out.

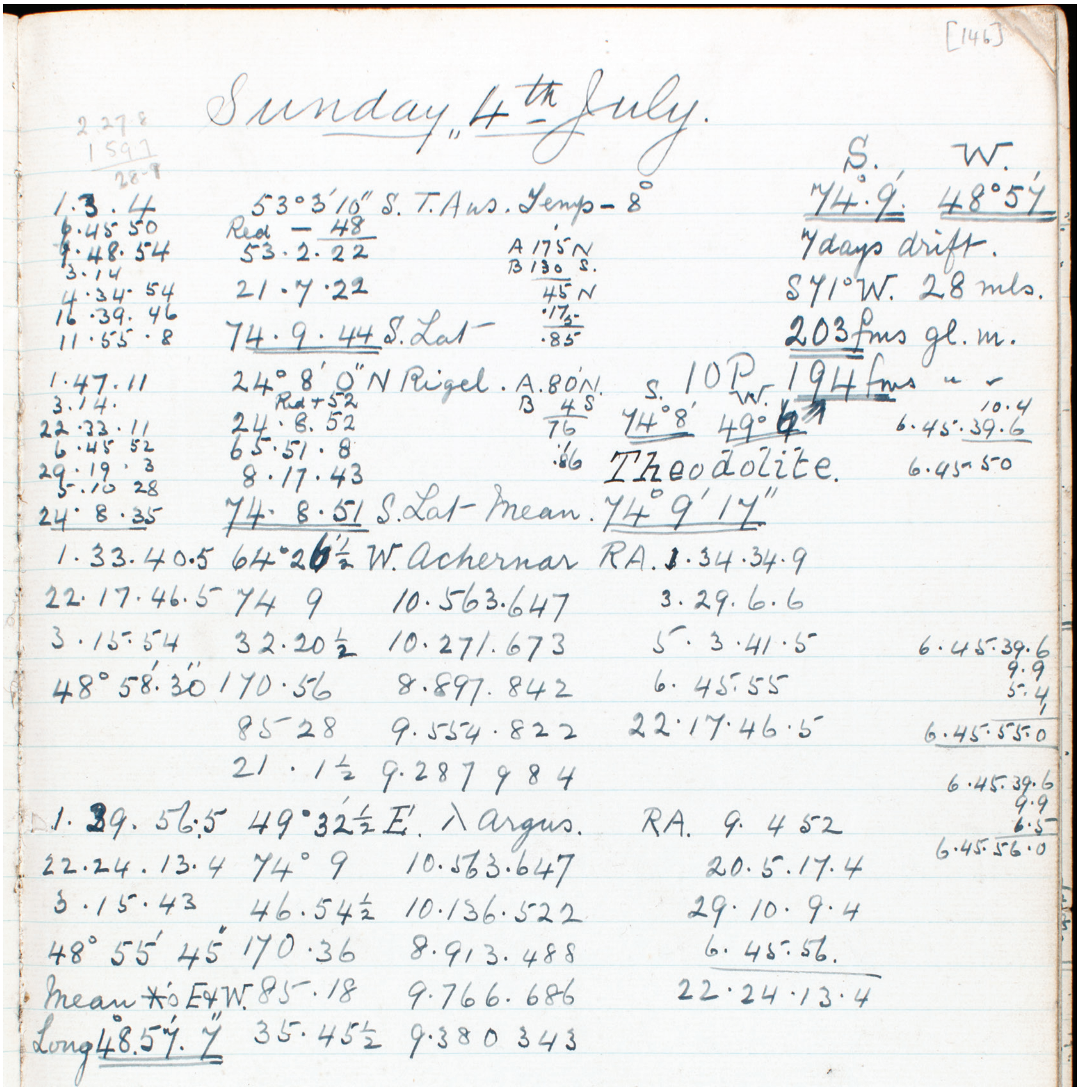

Figure 3. Log book entry for 4 July 1915 (Worsley 1915: 146) showing temperature, position, estimated drift and sounding data. Averaged ex-meridian sights of the stars α Trianguli Australis and Rigel determine latitude and averaged time sights of Achernar and λ Argûs (λ Velorum) provide longitude. Canterbury Museum 2001.177.1, page 146

Navigational Stars in the *Nautical Almanac*

In 1915, *The Nautical Almanac and Astronomical Ephemeris* (Nautical Almanac 1915) looked very different to the *Nautical Almanac* as it is known to practitioners of celestial navigation today. Tables were indexed with astronomical time with 0^h occurring at Greenwich Mean Noon on the day in question. The names of the brighter navigational stars are generally recognisable except for those in the defunct constellation of Argo Navis, which was officially split into Carina, Puppis and Vela by the International Astronomical Union (IAU) in 1930. In making the split, the original Greek letter designations from Argo were retained and not reallocated within the new constellations. For example, the second magnitude star λ Argûs today bears the designation λ Velorum (Suhail). Identifiers for fainter stars are selected from a hierarchy of catalogues and a large number bear the designation BAC, followed by a number of up to four digits. This refers to the British Association for the Advancement of Science Catalogue (British Association for the Advancement of Science 1845). The abbreviation B.D. refers to the Bonner Durchmusterung.

Sidereal Time

Local Sidereal Time (LST) is specified by the hour circle lying on the observer's meridian. Greenwich Sidereal Time (GST) is tabulated in the *Nautical Almanac* for Greenwich Mean Noon (0h) on each day of the year. GST advances at a rate of 9.8565 seconds per hour against Greenwich Mean Time (GMT), which is known as acceleration. At any given moment, GST − GMT = GST_0 + acceleration × GMT, where GST_0 is the tabulated value at the prior noon.

Ex-Meridian Sights for Latitude

If an object with declination, δ, is determined to have true altitude, h, at upper meridian transit, then the observer's latitude is $\phi = ZD + \delta$ where $ZD = 90° - h$ is the zenith distance. For a circumpolar object at lower meridian transit the observer's latitude is $\phi = h + p.d.$ in which $p.d. = 90° - \delta$ is the polar distance.

The Shackleton Expedition undertook ex-meridian sights of stars to determine their latitude. Dedicated tables, for example Brent et al. (1914), were available for this purpose but the Expedition instead used ABC tables, an example of which can be found in Lecky (1918: Part II Chapter IX). These were primarily designed to facilitate the computation of the azimuth of a body as might be required to determine the error in the ship's compass but they could be pressed into service to aid in the reduction of ex-meridian sights as well.

The azimuth of a body, Z_n, measured eastward from north is given by

$$\tan Z_n = \frac{-\sin t}{\cos\phi\tan\delta - \sin\phi\cos t}$$

where t and δ are the object's local hour angle and declination respectively and ϕ is the observer's latitude. Respecting the signs of the numerator and denominator of the right hand side ensures that the result for Z_n lies in the correct quadrant. ABC tables are constructed by defining

$$A = \frac{\tan\phi}{\tan t};\quad B = \frac{\tan\delta}{\sin t};\quad C = A + B \qquad (1)$$

and

$$\tan Z = \frac{1}{C\cos\phi}$$

Following nautical practice, rules are provided that assign names, N or S, or signs, + or −, to the quantities A, B, C and how to relate Z to Z_n. When t is near 0° or 180°

$$\frac{1}{C} = \cos\phi\tan Z \approx \cos\phi\sin Z = \left.\frac{dh}{dt}\right|_Z$$

where h is the object's true altitude with all quantities being taken to be positive. Let Δt be the difference of the local hour angle from the meridian of transit and Δh be the correction to be added to (subtracted from) h for upper (lower) transit. Assuming the ex-meridian altitude correction takes the quadratic form, $\Delta h = a \times (\Delta t)^2$, for some constant, a, then it is a simple matter to show that

$$\Delta h = \frac{1}{2}\left.\frac{dh}{dt}\right|_Z \Delta t$$

If Δt is given in units of time then Δh in arc is $\Delta h = (7.5/C) \times \Delta t$. The rules for combining A and B follow from the algebraic properties of equation (1) and imply that for circumpolar stars they should be added together for lower meridian transits and subtracted for upper transits. For all ex-meridian sights found in the log book, A is almost always assigned the name "N", B the name "S" and the two are subtracted one from the other. For most lower transits this biases the observed latitude to the north. When averaged between upper and lower transits the error in latitude is overall less than a nautical mile and generally much smaller.

Table 2 replicates the reduction of the ex-meridian sights of αTrianguli Australis and Rigel for latitude on 4 July 1915 using the altitude, Greenwich Mean Time and declination taken from the log. No great precision is required in the ex-meridian adjustment and it is clear that no great care was taken. In the log an error is made in the GST of the Rigel reduction and the *A* and *B* values are not the same as those shown here. Nevertheless the final result differs by only 20" or a third of a nautical mile.

Time Sights for Longitude

Longitude is found by comparing the observer's Local Mean Time (LMT) to GMT as read from a chronometer. A time sight measures the altitude or zenith distance of a celestial body sufficiently far off the meridian and from which its local hour angle (LHA) can be calculated. For a star at right ascension, R.A., the local sidereal time is then LST = LHA + R.A. The difference (GST – GMT) can be obtained by consulting tables in the *Nautical Almanac* as described previously and LMT = LST – (GST – GMT).

Table 3 shows the reduction of time sights of Achernar to the west and λ Argûs (λ Velorum) to the east replicated using values found in the log. The layout and labelling closely follows that described in Bergman et al. (2018) to which the reader is referred for details.

S. α Trianguli Australis (Below Pole)

Mean time at Greenwich	1^{h}	3^{m}	4^{s}	Altitude	53°	3′	10″	A	167 N
GST - GMT	6	45	50	Reduction		–	9″	B	122 N
GST	7	48	54	Meridian altitude	53	3	1	C	289 N
Longitude in time	3	14		Polar Distance	21	7	22	7.5/C	0.03
LST	4	34	54	Latitude	74°	10′	23″ S.	Δt	5^{m}
Right Ascension	16	39	46					Reduction	0.15′
Local Hour Angle	11	55	8					=	9″
				GST at Noon	6^{h}	45^{m}	39.6^{s}		
				acceleration			10.4		
				GST – GMT	6	45	50		

N. Rigel

Mean time at Greenwich	1^{h}	47^{m}	11^{s}	Altitude	24°	8′	0″	A	94 N
GST - GMT	6	45	57	Reduction		+	52″	B	4 S
GST	8	33	8	Meridian altitude	24	8	52	C	90 N
Longitude in time	3	14		Meridian ZD	65	51	8	7.5/C	0.1
LST	5	19	8	Declination	8	17	43	Δt	8.7^{m}
Right Ascension	5	10	28	Latitude	74°	8′	51″ S.	Reduction	0.87′
Local Hour Angle	24	8	40					=	52″
				GST at Noon	6^{h}	45^{m}	39.6^{s}		
				acceleration			17.6		
				GST – GMT	6	45	57.2		

Mean ✶'s N &S. 74° 9′ 37″ S.

Table 2. Reduction of ex-meridian sights of α Trianguli Australis and Rigel for latitude on 4 July 1915. The results are averaged to mitigate uncertainties in refraction.

W. Achernar													
Mean time at Greenwich	1^{h}	33^{m}	41^{s}	Altitude	64 °	26.5 ′	AM		R.A.		1^{h}	34^{m}	34.9^{s}
Mean time at ship	22	17	46.5	Latitude	74	9.0	sec.	0.563647	LHA		3	29	6.6
Longitude in time	3	15	54	Polar distance	32	20.5	cosec.	0.271673	LST		5	3	41.5
Longitude	48 °	58 ′	30 ″	Sum	170	56.0			GST – GMT		6	45	55.0
				Half-sum	85	28.0	cos.	8.897842	LMT		22	17	46.5
				Remainder	21	1.5	sin.	9.554822					
							hav.	9.287984					
									GST at Noon		6^{h}	45^{m}	39.6^{s}
										1 h			9.9^{s}
									acceleration	33 m			5.4^{s}
										40.5s			0.1^{s}
									GST – GMT		6^{h}	45^{m}	55.0^{s}
E. λ Argûs													
Mean time at Greenwich	1^{h}	39^{m}	57^{s}	Altitude	49 °	32.5 ′	AM		R.A.		9^{h}	4^{m}	52.0^{s}
Mean time at ship	22	24	13.3	Latitude	74	9.0	sec.	0.563647	LHA		20	5	17.4
Longitude in time	3	15	43	Polar distance	46	54.5	cosec.	0.136522	LST		29	10	9.4
Longitude	48 °	55 ′	45 ″	Sum	170	36.0			GST – GMT		6	45	56.1
				Half-sum	85	18.0	cos.	8.913488	LMT		22	24	13.3
				Remainder	35	45.5	sin.	9.766686					
							hav.	9.380343					
Mean ✶'s E &W.	48 °	57 ′	7 ″						GST at Noon		6^{h}	45^{m}	39.6^{s}
										1 h			9.9^{s}
									acceleration	39 m			6.4^{s}
										56.5s			0.2^{s}
									GST – GMT		6^{h}	45^{m}	56.1^{s}

Table 3. Reduction time sights of Achernar and λ Argûs (λ Velorum) for longitude on 4 July 1915. The results are averaged to mitigate uncertainties in refraction.

Longitude by Occultation of a Fixed Star

As recounted by Reginald James (Shackleton 1920: Appendix 1):

> *During the voyage of the Endurance about fifteen months elapsed during which no check on the chronometers could be obtained by the observation of known land, and had no other check been applied, there would have been the probability of large errors in the longitudes.… In the summer, however, the [occultation] method is quite impossible since, for some months, stars are not to be seen.*
> *No chronometer check could be applied until June, 1915.*

In the depths of the polar night and a few days after the winter solstice, the first such occultation timing was made on 24 June of the star 42 Libræ followed by three others on the same astronomical date. These were averaged to rate the Mercer chronometer, No. 5229.

For a position of known latitude, noting the local mean time (LMT) or local sidereal time (LST) of immersion or emersion in an occultation allows the GMT or equivalently longitude to be found.

A few methods are available for this. Bessel's method projects the positions of the observer and Moon orthographically onto the so-called fundamental plane passing through the Earth's centre and with its normal in the direction of the star. This is effectively the viewpoint of an observer located on the star. The calculation is eased by the Besselian elements, T_0, q_0, p' and q' tabulated in the *Nautical Almanac* which incorporate all necessary information related to the Moon's horizontal parallax (HP) and semi-diameter. This approach results in a quadratic equation that can be solved by means of some judicious trigonometric substitutions (Chauvenet 1863: 550).

Raper's Method

The Expedition used a method detailed by Close (1905), which comes from the Royal Geographical Society's publication, *Hints to Travellers* (Godwin-Austen et al. 1883) and is attributed to Raper (1840). By accounting for the effect of parallax, the geocentric right ascension of the Moon can be determined at the moment

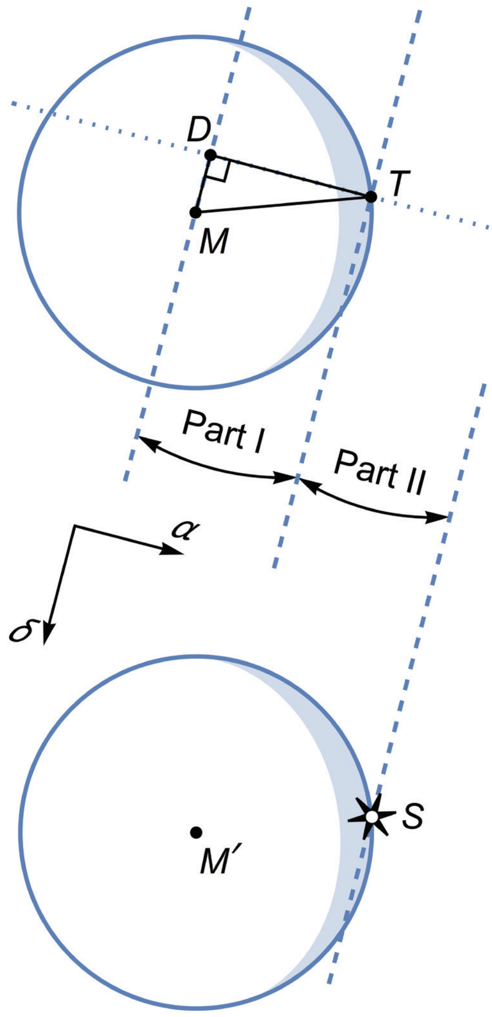

Figure 4. The occultation of 42 Libræ on 24 June 1916. The lower circle centred on M' represents the Moon as seen by the Shackleton Expedition in the Weddell Sea and the upper circle represents the view of a geocentric observer. The star disappears at the point S on the Moon's limb which corresponds to point T for the geocentric observer. Directions of increasing right ascension and declination are indicated by the axes labelled α and δ. The dashed lines are hour circles and the dotted line indicates the diurnal circle of constant declination passing through the point T.

the occultation occurs. Comparing this with tables of the Moon's position in the *Nautical Almanac* gives GMT.

Figure 4 depicts the occultation of 42 Libræ as it was observed from the Weddell Sea at 4^h44^m GMT in astronomical time or 16^h44^m civil time on 24 June 1915. At a faint magnitude of 5.1, 42 Libræ stood SE by E around 15° above the horizon before blinking out behind the dark limb of a waxing gibbous Moon two and a half days from full. The circle centred on the point M' represents the Moon's apparent position as seen by the Expedition with the point on its limb where the star disappeared labelled S. The axes labelled α and δ show the directions of increasing right ascension and (northerly) declination respectively. The upper circle centred on the point M is the Moon's disk as it would appear to a geocentric observer displaced by parallax toward the geocentric zenith. For such an observer, the position of the star is unchanged due to its great distance from Earth, and the point S on the Moon's limb now lies nearly vertically above at the point labelled T. The declination of T is the Prepared Declination. The dashed lines are hour circles of right ascension passing through S, T and M that converge to meet at the poles. The dotted line is the diurnal circle of constant declination passing through T. The angle subtended at the pole by the arc labelled Part II is the parallax in right ascension. Part I in the diagram subtends the difference in right ascension of the Moon's centre and the point T. The distance MT is the Moon's geocentric semi-diameter.

Starting from the star's R.A., Part I and Part II allow the geocentric R.A. of the Moon's centre to be computed at the time of immersion and hence GMT to be determined.

For the calculation of parallax, the relative positions of the Earth, Moon and observer need to be specified in 3-dimensional space taking account of the oblateness of the Earth. As is explained in standard texts (Smart 1948) the observer's position is specified in terms of the geocentric distance, ρ, and geocentric latitude, ϕ'. The latter differs from the astronomical latitude, ϕ, measured in celestial navigation. If a is the Earth's equatorial radius and r is the Moon's geocentric distance, then the equatorial horizontal parallax, HP, is defined as sin HP $= a/r$. Since $\rho/r = (\rho/a)\sin$ HP it is intuitively plausible, and can be shown rigorously, that the impact of the observer's geocentric distance can be absorbed into a reduced or local horizontal parallax, HP$'$, satisfying the condition $\sin HP' = (\rho/a)\sin HP$.

Let t' and δ' denote respectively the apparent local hour angle and declination of a point S on the Moon's face as seen by an observer on the Earth's surface. It is required to compute the true hour angle, t, and declination, δ, of that point, displaced by parallax and labelled point T, as it would be seen by a geocentric observer. A derivation of the following exact formulas can be found, for example, in Loomis (1855)

$$\sin(t'-t) = \sin HP' \frac{\cos\phi' \sin t'}{\cos\delta} \tag{2}$$

$$\sin(\delta-\delta') = \sin HP' \sin\phi' \cos\delta' - \sin HP' \cos\phi' \sin\delta' \frac{\cos\frac{1}{2}(t'+t)}{\cos\frac{1}{2}(t'-t)} \tag{3}$$

Writing $\frac{1}{2}(t'+t)=t'-\frac{1}{2}(t'-t)$ leads to

$$\sin(\delta-\delta') = \sin HP' \sin\phi' \cos\delta' - \sin HP' \cos\phi' \sin\delta' \cos t' - \sin HP' \cos\phi' \sin\delta' \sin t' \frac{\sin\frac{1}{2}(t'-t)}{\cos\frac{1}{2}(t'-t)} \tag{4}$$

With $\cos\frac{1}{2}(t'-t)\approx 1$ and $\sin\frac{1}{2}(t'-t)\approx\frac{1}{2}\sin(t'-t)$, equation (4) can be used to obtain

$$\sin(\delta-\delta') = \sin HP' \sin\phi' \cos\delta' - \sin HP' \cos\phi' \sin\delta' \cos t' - \frac{1}{2}\left(\sin HP' \cos\phi' \sin t'\right)^2 \frac{\sin\delta'}{\cos\delta} \tag{5}$$

In the approximation that $\sin\theta \approx \theta$ for small θ and $\sin\delta'/\cos\delta \approx \tan\delta'$, expressed in degrees equation (5) becomes

$$\delta = \delta' + \mathrm{HP}'\sin\phi'\cos\delta' - \mathrm{HP}'\cos\phi'\sin\delta'\cos t' - \frac{1}{2}\left(\mathrm{HP}'\cos\phi'\sin t'\right)^2 \tan\delta' \frac{\pi}{180} \tag{6}$$

As noted earlier declination of the point *T*, δ, is called the prepared declination and is obtained by means of equation (6).

If α' is the right ascension of point *S* then, to sufficient accuracy, by equation (2) the right ascension, α, of the point *T* in degrees is

$$\alpha = \alpha' + \mathrm{HP}'\frac{\cos\phi'\sin t'}{\cos\delta} = \alpha' + \text{Part II} \tag{7}$$

where Part II is the parallax in right ascension.

Let $\alpha_☽$, $\delta_☽$ and SD denote the Moon's geocentric R.A., declination and semi-diameter respectively. In Figure 4, the right triangle *MDT* is small enough that it can be adequately treated using plane trigonometry and hence $(DT)^2 = (MT)^2 - (DM)^2 = SD^2 - (\delta - \delta_☽)^2$. The difference in R.A. between the points *D* and *T* is then

$$\text{Part I} = \frac{\sqrt{\left(\mathrm{SD} - \delta + \delta_☽\right)\left(\mathrm{SD} + \delta - \delta_☽\right)}}{\cos\delta} \tag{8}$$

The geocentric R.A. of the Moon at the moment the occultation occurs is then

$$\alpha_☽ = \alpha' \pm \text{Part I} + \text{Part II} \tag{9}$$

where the upper (lower) sign applies for emersion (immersion). In the 1915 *British Nautical Almanac* (Nautical Almanac 1915), the Moon's geocentric position is tabulated at hourly intervals and the time at which the GMT corresponding to $\alpha_☽$ is computed by simple interpolation. If α_0 is the tabulated value of the Moon's R.A. at the hour t_0 preceding the occultation and α_1 is its R.A. on the hour following then required time is

$$\mathrm{GMT} = t_0 + \frac{\left(\alpha_☽ - \alpha_0\right)}{\left(\alpha_1 - \alpha_0\right)} = t_0 + x \tag{10}$$

The foregoing equations are algebraically fully correct but in traditional navigational practice quantities are generally considered to be positive. Rules are given as to when to add or subtract terms based on conditions, such as whether certain dependent variables have the same or contrary name, i.e. the same or opposite sign. In the case of equation (6) the three last terms on the right hand side are labelled Arc A, Arc B and Arc C respectively and are combined with δ' to obtain the geocentric declination of point *T*, δ.

Raper (1840) describes, without derivation, the procedures to be followed to obtain the geocentric declination:

> *(4.) When the lat. and decl. are of the same name, add **A** to the star's decl. ; when of contrary names, subtract it.*
> *When the star's hour-angle is less than 6^h^, subtract **B** from the star's decl. ; when greater than 6^h^, add it.*
> *Subtract **C** from **A**.*
> *Call the result the prepared declination.*

The rules are deduced from the algebraic properties of equation (6). In the example included by Raper, it is more or less clear that his intended meaning for the last rule would be better stated as simply 'Subtract ***C***'. *Hints to Traveller's* (Godwin-Austen et al. 1883) first gives an example of finding longitude by the occultation of a fixed star using "Raper's rule and tables" in which the difference (***A***–***C***) is computed first and then added to or subtracted from δ' according to the rule for ***A*** alone. Reeves (1904) derives equation (6) but does not correct the rule for Arc C which continues unchanged in *Hints to Traveller's* under his editorship (Reeves 1906). The value of Arc C is generally small but in principle could be as large as 17". Incorrect results are produced only for the case when declination and latitude have contrary names. The Shackleton Expedition inherited this erroneous methodology through Close (1905). Although the Moon spends half of its time in the northern hemisphere and half in the southern, the local circumstances of an occultation are more favourable for an observer in the polar regions when latitude and declination have the same name and hence the error does not manifest itself.

Proportional Logarithms

In his description of the calculation, Raper employs common logarithms for trigonometric functions and proportional logarithms or prologs for other quantities. These were originally introduced by Maskelyne (1781) as a means of simplifying the interpolation computing Greenwich time from lunar distances. The *Nautical Almanac* tabulated geocentric lunar distances at three hour intervals and for an argument in hours or degrees, the ternary proportional logarithm is defined as $\text{Prolog}(x)=\log_{10}(3/x)$. Their use here, however, offers no clear advantage over common logarithms and requires that trigonometric functions be replaced by their reciprocals.

The calculations demonstrated in *Hints to Traveller's* (Godwin-Austen et al. 1883) use four-figure logarithms, which continues up to and including the eighth edition. Both Reeves (1906) and Close (1905) give examples using five-figure logarithms but truncate the constants that enter at four digits. These truncated constants are inherited by the Expedition.

Lunar Occultations of 24 June 1915

Figures 5a and 5b show the pages where the reduction of the occultation of 42 Libræ is performed. This is the first occultation and the steps are carefully labelled.

An initial reduction of the occultation is not recorded. It changes the CE of the Mercer (5229) chronometer from $6^m\ 24^s$ fast of GMT, as entered in the log prior to the occultation, to $1^m\ 38^s$ fast. This result is used as input to the second reduction, which is seen in the log and gives a CE of $2^m\ 2.8^s$ fast. It is followed by a note "reworked with $2^m3^s = 2^m0^s.4$" indicating that a third iteration was performed.

Timings and reductions of three additional occultations were made with the CE and longitude obtained from that of 42 Libræ being used as a starting point for these subsequent reductions.

The table in the bottom of Figure 5a indicates that at least on this occasion Worsley and James worked independently on reducing the occultations. The values in the column labelled "by Capt W." are the same ones found later in the log, leaving no doubt who had carried them out. The final CE of $2^m\ 1.7^s$ fast is the average over all observations and becomes the chronometer's new working error.

Due to the limited rate of change of the Moon's right ascension, the chronometer error can in this case only be determined to a precision of ¼ second, in worst cases no better than ⅓ second. Thus the use of two decimals is unnecessary.

The difference between $6^m\ 24^s$ and $2^m\ 1.7^s$ is $4^m\ 22^s$, corresponding to a longitude shift of 1°5'30" to westward, or 18 nautical miles (33 km) in departure. This is the "error in longitude of a whole degree" described by James (Shackleton 1920: Appendix 1).

Considering the last reliable rating was made 242 days earlier, the error in longitude is actually remarkably small.

In principle the LMT used in the reduction of an occultation should be corrected for changes in the observer's longitude since the time sight was made. However, owing to the speed at which the Moon's shadow sweeps across the Earth's surface, the GMT determined from the occultation is relatively insensitive to the exact value of LMT that is input. For example, in the reduction of the 42 Libræ, an error of 1 nautical mile in the estimated east-west position at these latitudes corresponds to a 15 second error in LMT, but leads to only a 1 second error in the value of GMT obtained.

Table 4a replicates the data preparation recorded in the log for the second iteration using input values found in the log and following Close (1905). It consists of computing geocentric latitude and interpolating the tabulated values of the Moon's R.A., declination, HP and semi-diameter at the time of the occultation. Corrections are applied to the mean position of the star to account for precession, proper motion and annual aberration. LST is computed and from it the LHA for the star.

Worsley follows the advice of Close (1905: 192) and diminishes the Moon's calculated semi-diameter "in proportion of 15'34".09 to 15'32".65

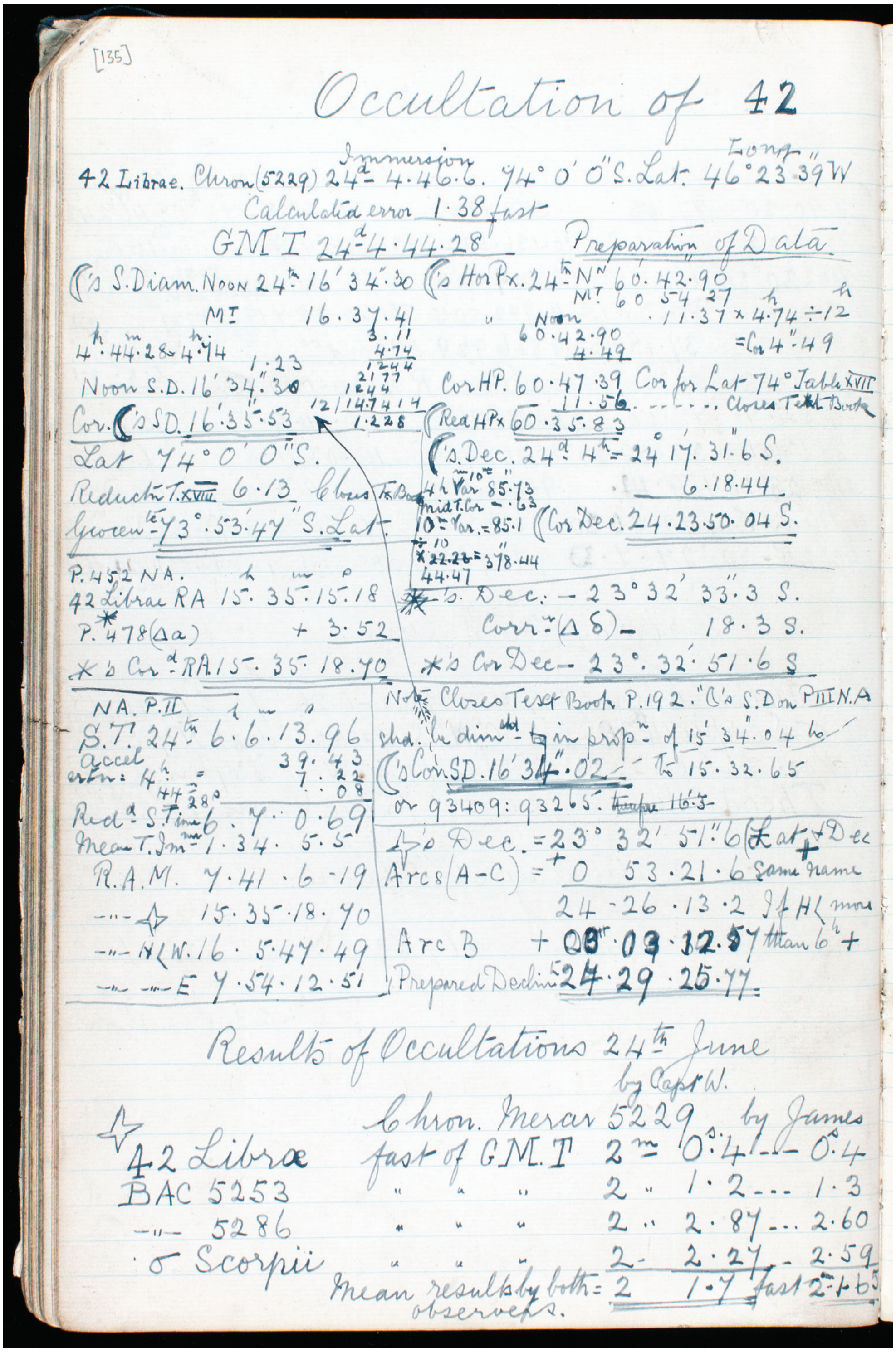

Figure 5a. First of two logbook pages (Worsley 1915: 135) showing the reduction of the occultation of 42 Libræ observed on 24 June 1915. Canterbury Museum 2001.177.1, page 135.

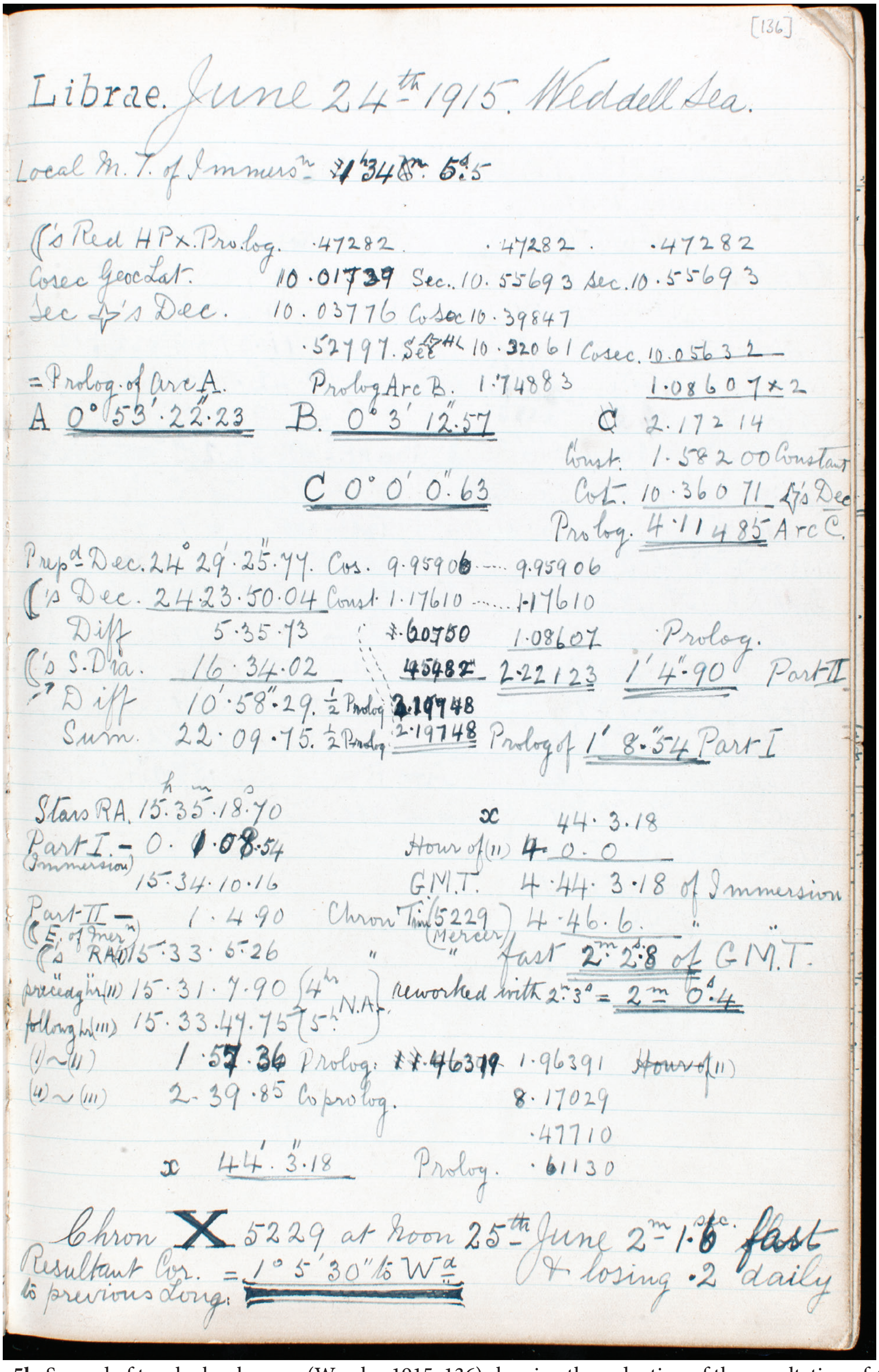

Figure 5b. Second of two logbook pages (Worsley 1915: 136) showing the reduction of the occultation of 42 Libræ observed on 24 June 1915. Canterbury Museum 2001.177.1, page 136.

Occultation of 42 Libræ	**24 June 1915**					
		h.	m.	s.		
Chron. Time of	Immersion	4	46	6.0		
Watch Error +Slow –Fast	–	0	1	38.0		
Corrected Greenwich Date		4	44	28.0		
Mean Time at Place		1	34	5.5		
Approx. Longitude	W.	46°	23′	39″		
		°	′	″		
Latitude	S.	74	0	0		
Reduction		0	6	13.2	(Table XVIII.)	
Geocentric Latitude (ϕ')	S.	73	53	46.8		
			′	″		
☽'s Semi-diameter, preceding			16	34.30	(p.III., \`N.A.')	
Ditto , following			16	37.41		
12-hourly difference			0	3.11		
Difference to 4h 44m 28s			0	1.23		
			16	35.53		
Irradiation adjustment				0.998	(93265/93409)	
☽'s Semi-diameter at Greenwich Date (SD)			16	33.99		
			′	″		
☽'s Hor. Parall. 24th Noon			60	42.90	(p.III., \`N.A.')	
Ditto Midnight			60	54.27		
12-hourly difference			0	11.37		
Difference to 4h 44m 28s			0	4.49		
☽'s Hor. Parall. at Greenwich Date			60	47.39		
Correction for Latitude			0	11.45	(Table XVII.)	
☽'s Reduced Horizontal Parallax (HP′)			60	35.95		
		°	′	″	Var. in 10^m	
☽'s Declination, preceding	S.	24	17	31.6	85.73″	(p. X., \`N.A.')
Ditto , following	S.	24	26	0.9	84.03″	
Correction		0	6	18.41	85.10″	
☽'s Reduced Declination (δ)	S.	24	23	50.01		

Table 4a. Preparatory data for the reduction of the occultation of 42 Libræ observed on 24 June 1915. Canterbury Museum 2001.177.1, page 135.

			h.	m.	s.	
Star's R.A.			15	35	15.18	(a)
Correction		+			3.52	(b)
			15	35	18.70	

			h.	m.	s.	
Sidereal Time of G.M. Noon			6	6	13.96	(p.II., `N.A.')
	4h.		0	0	39.43	
Acceleration	44m.		0	0	7.23	
	28s.		0	0	0.08	
Reduced Sidereal Time			6	7	0.69	
Mean Time at Place			1	34	5.50	
R.A. of Meridian			7	41	6.19	
Ditto Star			15	35	18.70	
Star's Hour Angle, (t')			7	54	12.51	E.
			118°	33′	7.64″	(in arc)

		°	′	″	
Star's Declination	–	23	32	33.3	(a)
Correction	–			18.3	(b)
	–	23	32	51.6	

		°	′	″
Star's Declination, (δ')	–	23	32	51.6
Arc A	–	0	53	22.39
Arc B	–	0	3	12.58
Arc C	+	0	0	0.83
Prepared Declination, (δ)	S.	24	29	25.74

(a) "Mean Places of Occultation Stars," `N.A.'
(b) "$\Delta\alpha$, $\Delta\delta$" in "Elements of Occultation," `N.A.'

Table 4a. (continued)

Prolog(HP′)	0.47281			
$\log_{10}\csc(\phi')$	10.01738			
$\log_{10}\sec(\delta')$	10.03776			
Prolog(Arc A)	0.52795			
Arc A		0°	53′	22.39″
Prepared Declination, δ		24	29	25.74
$\delta_☽$		24	23	50.01
		0	5	35.73
SD		0	16	33.99
Difference		0	10	58.26
Sum		0	22	9.72
		h.	m.	s.
Star's R.A.		15	35	18.70
-		0	1	8.538 (Part I., –, Immersion)
		15	34	10.162
-		0	1	4.895 (Part II., –, Moon is E. of Meridian)
☽'s R.A. (i.)		15	33	5.267
Ditto preceding hour (ii.)		15	31	7.90 (4 h, p. X., of 'N.A.')
Ditto following hour (iii.)		15	33	47.75 (5 h, p. X., of 'N.A.')
Difference between (i.) and (ii.)		0	1	57.367
Ditto (ii.) and (iii.)		0	2	39.85

Prolog(HP′)	0.47281
$\log_{10}\sec(\phi')$	10.55693
$\log_{10}\csc(\delta')$	10.39847
$\log_{10}\sec(t')$	10.32061
Prolog(Arc B)	1.74882
Arc B	0° 3′ 12.58″
$\log_{10}\cos(\delta)$	9.95906
$\log_{10}15$	1.17609
½Prolog	0.60751
½Prolog	0.45483
Prolog(Part I)	2.19749
Part I	$0^h\ 1^m\ 8.538^s$
Prolog	1.96388
Complement	8.17029
$\log_{10}3$	0.47712
Prolog(x)	0.61129
x	$0^h\ 44^m\ 3.23^s$

Prolog(HP′)	0.47281
$\log_{10}\sec(\phi')$	10.55693
$\log_{10}\csc(t')$	10.05632
	1.08606
	× 2
	2.17212
$\log_{10}(120/\pi)$	1.58203
$\log_{10}\cot(\delta)$	10.36071
Prolog(Arc C)	4.11486
Arc C	0° 0′ 0.83″
$\log_{10}\cos(\delta)$	9.95906
$\log_{10}15$	1.17609
	1.08606
Prolog(Part II)	2.22121
Part II	$0^h\ 1^m\ 4.895^s$

	h.	m.	s.	
Hour of (ii.)	4	0	0	
x	0	44	3.23	
GMT	4	44	3.23	
LMT	1	34	5.50	
	3	9	57.73	W.
(in arc)	47°	29′	25.90″	W.
Chron. Time	4^h	46^m	6.0^s	
Error		2^m	2.8^s	fast

Table 4b. Calculation of Greenwich Mean Time from the Local Mean Time of the occultation of 42 Libræ on 24 June 1915.

or 93409 : 93265". The semi-diameter tabulated in the *Nautical Almanac* includes adjustment for irradiation, defined as "an optical effect of contrast that makes bright objects viewed against a dark background appear to be larger than they really are" (Seidelman 1992: 729). This needs to be omitted when reducing occultations.

Unlike Close (1905) who uses linear interpolation for the Moon's declination, Worsley follows the prescription from the Explanation of the Articles section of the *Nautical Almanac* (1915: 619) and performs quadratic interpolation. For the right ascension (R.A.) and declination of the Moon, which change relatively rapidly, the *Nautical Almanac* tabulates values hourly and provides a column headed "Var. in 10^m" which permits quadratic interpolation between the values. Suppose the value of a quadratic function, $y(t)$, takes the value y_0 at $t = 0$ and its derivative takes the values y'_0 and y'_1 at $t = 0$ and $t = 1$ respectively. It is a simple matter to show that

$$y(t) = \tfrac{1}{2}\left(y_1' - y_0'\right)t^2 + y_0't + y_0$$
$$= \left\{ y_1'\frac{t}{2} + y_0'\left(1 - \frac{t}{2}\right)\right\} t + y_0$$

The factor in curly brackets is the linearly interpolated value of y' at time $t/2$. To paraphrase the almanac (Nautical Almanac 1915: 619) the prescription to compute the Moon's declination at an intermediate time is:

> *Reduce the "Var. of Dec. in 10m" to the time midway between the time for which the Declination is required and the preceding hour in the Ephemeris, and then obtain the correction by simple proportion.*

The reduced horizontal parallax and geocentric latitude that account for the figure of the Earth is read from tables XVII and XVIII in Close (1905). These are computed based on "Clarke's first figure (1858)" geodetic reference ellipsoid (Close 1905: 209) with compression, or what is today called flattening, f = 1/294.26. For subsequent occultations, however, the log states "Using compress$^{\underline{n}}$ Red$^{\underline{n}}$ of Earths Polar Axis 1/293.47 Clarke 1866".

Table 4b shows the calculation of GMT from the observed LMT of immersion 42 Libræ again based on the layout given by Close (1905), but with extended annotation in some areas. Constants have been extended to full five-figure accuracy. Small differences are seen from the numbers that appear in the log mainly where interpolation has been performed. In order to avoid negative numbers, it was standard practice to add 10 to the logarithms of trigonometric functions. In many cases it is done even when strictly unnecessary, such as for secants and cosecants. In the sum of logarithms, digits in the tens column are discarded.

Excerpts of pages from the *Nautical Almanac* (1915) consulted for these inputs are given in Appendix B.

'A Very Good Series of Occultations'

Following this first round of occultations on 24 June, further sets of observations were carried out and are listed in Table 5. All observations were of immersion on the dark limb of a waxing Moon which are more reliably timed than emersion or immersion on a bright limb. With the exception of σ Scorpii all stars are relatively faint at 5th and 6th magnitude requiring that the Moon be at a reasonable altitude for them to be visible. At the latitudes concerned there are periods of several days in each synodic month when the Moon is very low or completely below the horizon. These considerations along with the constraints of weather and periods of increasing sunlight limit the availability of events suitable for timing.

The log records a longitude for each occultation but this does not enter in the reduction and does not always agree with the longitude inferred by subtracting LMT from GMT.

Intermediate calculations from the reduction of the occultation of BAC 4722 on 16 August 1915 are found on a loose leaf page (Canterbury Museum 2001.177.24, loose page).

The CE obtained from the occultations was used to correct noon longitudes from prior dates which are crossed out and adjusted accordingly.

On an unused page opposite the log entries for 1 January 1915 (Worsley 1915: 57) is written:

> *Starting from June1st 1915 the error of A. Chronometer (2235) Heath & Co. was assumed to be 29 seconds more slow than had been allowed. This was calculated back from a very good series of occultations from June 24th to Sept. 15th 1915. The changing rates were plotted on a curve by Mr Hudson Navigtg Officer & from this the corrections for longitude were deduced; as being at a maximum on June 24th & tapering down to zero at Buenos Ayres 24th Oct. 1914.*

Worsley (1915: 73) summarised the CE and rate as deduced from the occultations in an entry facing the log page of 18 March 1915. These are shown in Table 6.

Reginald James writes "After the crushing of the ship on October 27, 1915, no further occultations were observed" (Shackleton 1920: Appendix I).

Navigation at Ocean and Patience Camps

Navigation during the period camped on the ice consisted of noon sights and time sights of the Sun as well a frequent updates of the distance and bearing of potential target destinations.

In late March 1916, their position approached Joinville Island. Its highest peak, Mount Percy, with position taken from a chart in Nordenskjöld (1905) as 63°14'S 55°38'W (Worsley 1916: 4), could provide a means to rate chronometers and fix longitude.

On 23 March 1916 (Worsley 1916: 83) wrote the following and included an illustration of

Star	HIP	Magnitude	Altitude of Star	Age of Moon	Latitude	Greenwich Mean Time	Local Mean Time	Hour Angle
			°	days	° ′ ″	h m s	h m s	° ′ ″
24 June, 1915								
42 Libræ	76742	5.1	15	11.9	74 0 0 S.	4 44 28	1 34 5.5	118 33 8 E.
B.A.C. 5253	77858	5.4	35	12.1	73 58 0 S.	9 49 56	6 40 5	45 14 12 E.
B.A.C. 5286	78246	5.4	40	12.2	73 57 45 S.	11 44 46	8 34 55	17 36 55 E.
σ Scorpii	80112	3.1	19	12.6	73 57 0 S.	20 31 57	17 22 18	108 57 21 W.
23 July, 1915								
A Ophiuchi	84405	5.3	10	11.1	73 14 30 S.	0 36 18	21 24 8	176 20 45 E.
16 August, 1915								
B.A.C. 4722	69658	5.6	37	5.9	70 41 43 S.	8 36 0	5 15 51	10 25 32 W.
B.D. -17°4053	69792	6.4	35	5.9	70 41 43 S.	9 33 40	6 13 23.5	24 26 18 W.
B.A.C. 4739	69929	5.7	34	6.0	70 41 43 S.	10 19 11.5	6 59 2.5	35 29 19 W.
13 September, 1915								
B.D. -21°4030	73927	6.1	25	4.5	69 45 0 S.	11 57 43	8 35 51.5	75 27 20 W.
15 September, 1915								
A Ophiuchi	84405	5.3	31	6.6	69 32 0 S.	13 43 13	10 19 52	71 21 14 W.

Table 5. Lunar occultations recorded in the logbook. Astronomical dates and times are listed. The columns Star and Magnitude give the designation and magnitude listed in the *Nautical Almanac* (1915). HIP is the Hipparcos Catalogue number. Latitude, Greenwich Mean Time, Local Mean Time and Hour Angle come from values given in the logbook.

position of Mount Percy (Fig. 6): "M^{t} Percy West 57 miles", "Appearance of land S60°W by comp. 9/0 A.M", "in this lat. the true bearing of a place 57 miles West wd be about S86°W."

A note, "Error 14°E ?", gives the assumed compass variation or magnetic declination:

> *From these bearings it wd appear as the Mt P (if that be the highest peak we see) is further N. than 63°14'S. as on Nordenskjold's chart or else that we are further S. by some error than our Obs. shew. The latter appears very unlikely.*

24 March (Worsley 1916: 85): "M^{t} Percy S89°W. [true] 60½ [miles]" as calculated by traverse tables "M^{t}Percy S74°W(mag) 0°11'" measured by theodolite.

Using a height of 3,673 feet (1,120 m) (Worsley 1916: 4) the measured altitude is used to calculate the distance to Mount Percy using the formula (Worsley 1915: 1): "Height in feet × .565 ÷ angle in ' = dist in miles"

The clearly erroneous result of 173 nautical miles (320 km) is due to the use of a simple proportions, which is not applicable for objects located beyond the visible horizon. Further bearings and vertical angles are taken the same day, S75°W magnetic giving S88½°W true with error 13½°E, 0°11' altitude; by theodolite S88°24'W.

Additional observations of Mount Percy are recorded on 26 and 27 March.

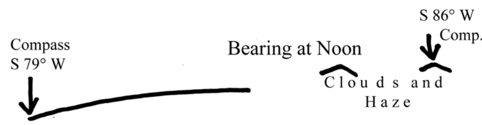

Figure 6. Copy of Worsley's sketch of the position of Mount Percy from the log entry of 23 March 1915.

On 24 March, Mount Percy had been observed bearing S88°24'W and, after drifting N23°E 16½ nautical miles (30.5 km), was measured at S73°38'W on 27 March. The distance to Mount Percy was found to be 57¼ nautical miles (106 km) on the latter date by triangulation, as shown in Figure 7. Worsley concluded (Worsley 1916: 80) that this is "10½^{m} = 23'W of Chron:" but that:

> *I will not allow any change to chron in meantime (Clarence or Elephant if seen soon will give better bearings & a good "fix") as had we been 10½ m West we wd have passed within 18m of the NE Danger in moderately clear weather, in wh: case we should almost certainly have seen it distinctly; whereas we only thought we saw it (It may have been hidden at times by grounded bergs. I do not know its height). Added to not having seen the Dangers or Darwin I, is the fact that the "chart" from N^{ds} book is evidently only intended for the general public & has not the close accuracy*

Chron (X) 5229 Mercer
18th March Error used 3^{m}56^{s}.5 fast Rate used 1.5 sec gaining
The errors were subsequently by Occultations to be as follows:-

Date			(X) 5229 Chron m s.		sec:	
18th March	1915	fast	1·36· 5	gaining	0.3	worked back from
24th June	–"–	–"–	2· 1·65	–"–	0.3	Occultations
23th July	–"–	–"–	2·13·25	–"–	0.4	
16th Aug	–"–	–"–	2·22·21	–"–	0.37	
13th Sept	–"–	–"–	2·25·08	–"–	0.1	

Table 6. Chronometer error and rate for the 5229 Mercer (Chron X) as obtained from the series of occultation timings, 24 June–13 September and those inferred for 18 March, 1915 (Worsley 1915: 73).

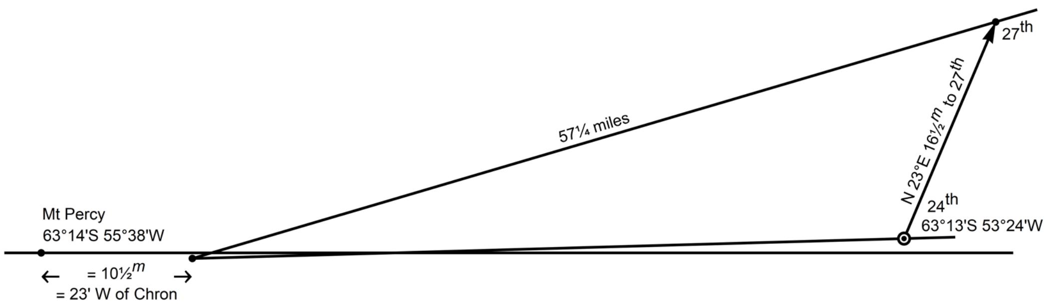

Figure 7. Triangulation of the position of Mount Percy on Joinville Island from its bearing angles on 24 and 27 March 1916 and the intervening drift.

> *of pos^tn for Mt Percy necessary to correct chrons – our great distance, small change of angle & a certain amount of doubt as to whether we have got the right point for Mt Percy*

James (Shackleton 1920: Appendix I) wrote that after the occultation timings ceased:

> *… calculated rates for the watches were employed, and the longitude deduced, using these rates on March 23, 1916, was only about 10' of arc in error, judging by the observations of Joinville Land made on that day.*

The logbook (Worsley 1916: 80) shows a graphical construction similar to Figure 7 (Bergman et al. 2018: Appendix A) but has the course angle from 24 to 27 March plotted incorrectly and is therefore likely to be a copy of the one actually used.

On 7 April 1916, Clarence Island was sighted and on 9 April did yield the sought after "good fix" (see Appendix A).

The Chronometers

Worsley (1998) states that *Endurance* set out carrying 24 chronometers. Of these, six are mentioned along with their letter designations, serial numbers, chronometer errors and rates for 24 June and 23 July 1915 (Canterbury Museum 2001.177.11, loose page front) (Fig. 8). The information is repeated for a selection of the chronometers on other dates on the reverse side. Loose leaf pages show daily comparisons with columns headed "Hudson 192/232 | Wild 192/231 | Worsley 192/262 | A" and others that are not distinguishable (Worsley 1916: 1-3).

The Mercer chronometer, No. 5229 or "Chron X" is frequently mentioned in the log suggesting that it was the primary one used. It now resides in the National Maritime Museum in Greenwich, United Kingdom (Object Id: ZAA0029)[2] and is believed to have been carried on Shackleton's famous boat journey on the *James Caird* from Elephant Island to South Georgia in 1916.

Particularly noteworthy is the Smith chronometer, serial number 192-262, that Worsley used in navigating the *James Caird* and is now in the collection of the Scott Polar Research Institute of Cambridge University, United Kingdom (Reference number: N: 999a)[3]. It makes its first appearance in the log on 2 November 1915 (Worsley 1916: 39). On this day its error is given as $40^m\ 26^s$ slow. On 7 November the error is given as $3^m\ 28.5^s$ fast apparently indicating that it had been reset. In the following weeks until 2 February 1916, entries on the daily log pages show it to be steadily losing, with a fluctuating rate of between 0 and 10 seconds per day indicating that it was being compared to another chronometer with a rate that was believed to be known.

A log page contains a list of comparisons from 3 February to 6 March 1916 (Worsley 1916: 3). Here it can be seen that Worsley's chronometer is losing 5 seconds per day, Hudson's 2 seconds

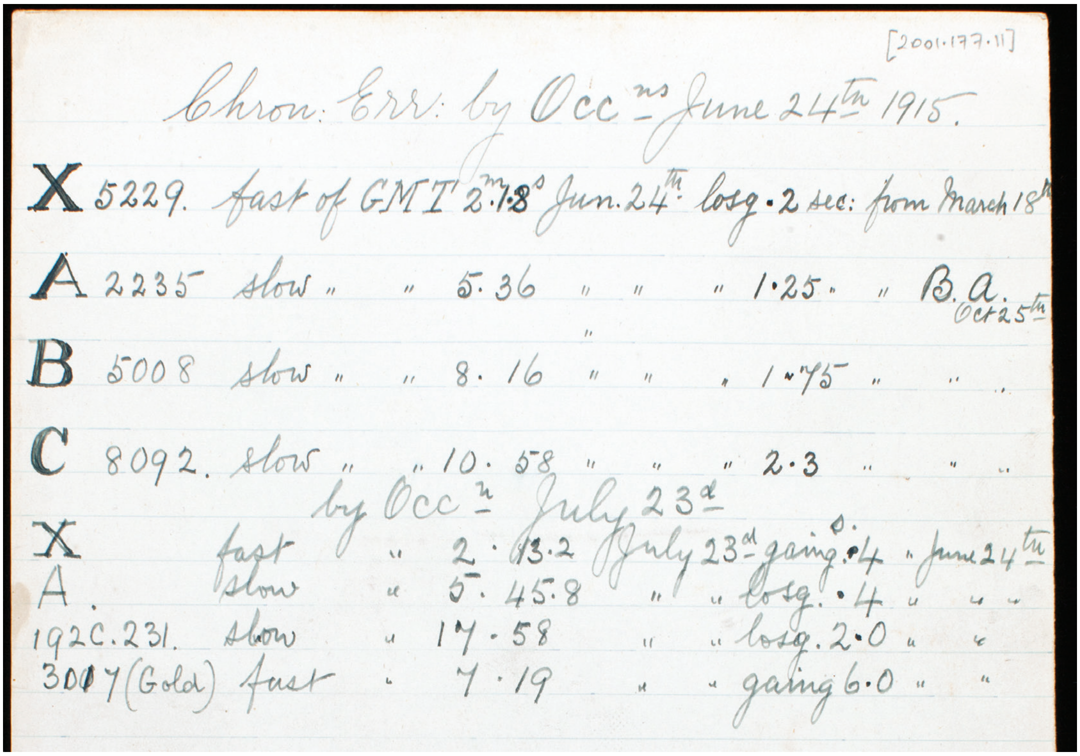

Figure 8. Some of the chronometers carried by the Expedition giving their letter designations, serial numbers, chronometer errors and rates. Canterbury Museum 2001.177.11, loose page front.

per day and Wild's 1 second per day. From 3 February onwards, Worsley calculates with a steady rate of 5 seconds per day (losing) and uses this value during the voyage of the *James Caird*.

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Endnotes

1 The standard nautical interpretation of gl.m. would be globigerina mud, however, it is clear from other commentary (Shackleton 1920; Wordie 1921) that the Expedition used it to indicate glacial mud or clay.

2 http://collections.rmg.co.uk/collections/objects/79134.html

3 http://www.spri.cam.ac.uk/archives/shackleton/articles/N:_999a.html

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Appendix A

The Passage to Elephant Island

This appendix describes the navigation performed during the passage from Patience Camp on the Weddell Sea to Elephant Island. Shackleton (1920) sighted Clarence Island on 7 April 1916 and later in the day Elephant Island was seen. Both were seen again the following day. The boats were launched at 1:30 pm on 9 April around 55 nautical miles from Elephant Island and the Expedition made landfall on Cape Valentine at its eastern tip on 15 April.

Time sights and the distance and bearing of Clarence and Bridgeman Islands are recorded prior to departure on 7 and 8 April. On both these days two observations were taken in the morning a few minutes apart, and when reduced with the same latitude gives nearly the same resulting longitude which provides assurance that the sights were "good". The last of these sights were timed and reduced to 0.1 seconds. In practice this is overkill in high latitudes but indicates that Worsley was striving for utmost accuracy in preparation for chronometer rating. The sight is, however, still reduced with five-figure logarithms although it might be expected that he still had six-figure tables in his possession. A bearing of Clarence Island taken around noon on 9 April was later used to rate the chronometers. No noon sights are recorded although they were taken. Most of the navigation underway is by Dead Reckoning with positions being determined by means of traverse tables. On 12 April time sights were made. On this day two morning sights were taken in rapid succession and reduced with different latitudes, again indicating that Worsley wanted to be sure of his position and gauge the magnitude of potential errors. Worsley (1998) recounts:

"....I took observation for longitude with the sextant. At noon I observed latitude....I thought we had made thirty miles towards Elephant Island. The sights proved we were thirty miles further away and had been driven nineteen miles farther south."

The entry for 13 April is labelled 13 Jan which may be an indication of the strain that Worsley was under at the time.

The navigational records of the passage are contained in just two detached log pages which, unsurprisingly considering the conditions under which they were used, are in relatively poor condition compared to much of the rest of the log (Worsley 1916: 89, 91). Numbers near the margins are often missing. In this appendix the discernible characters have been transcribed. Some of the missing values can be found in Worsley (1998) and are enclosed in curly brackets { }. Others can be calculated using the numbers that can be seen and are indicated with square brackets [ ]. Characters that cannot be read are denoted x.

Where possible, the log entries made under way replicated and were labelled using the notation and conventions established in Bergman et al. (2018). The distance and bearing calculations for 7 and 8 April are also replicated but the time sights have been omitted.

Transcript from the Log

Friday 7th April

slow 8 36

25. 5 20. O	14° 56 AM Sext		25.11 50	15 24 AM Theod			62° 8′0″	54°22′
- 2 13	62 8½	330.42	- 2 13	62 8½	330.42		2 days N 29° E 7m	
25 3 7	96 49½	3.09	25 9.37	96 49½	3.09	Cl.Pk	S 61°12′? N20° E 5[9m]	
21 25 43	173 54	725 97	21. 32 3	174 22	691.44		W53°40′	
[3] 37 24	86 57	978 24	3. 37 34	87.11	977.67	Bridgeman	S 88°W 5[8m]	
[5]4°21′	72 01	037 72	54°23′30″	71.47	002 62			

Saturday 8th April

slow 8 41

24 48 13 O	13° 27½AM Sext		25. 2 42. 5 ~~O~~	14°35½ AM Theod			62° 6′0″	53°4[8′]
- 1 56	62 6½	329.94	- 1 55.9	62 6½	329.94		N 82° E	16m
24.46.17	97 12	3.44	25. 0.46.6	97.12	3.44	Clarence	N 4°E[54 m]	
21 10 58.5	172. 46	799 90	21.25 35.5	173 54	725.97			
3 35 18.5	86 23	980 42	3.35 11.1	86.57	979.08			
53°49′37″	72 55½	113 70	53°47′50″	72 21½	038 43			

Thru mist — Sunday 9th April

may have been

Elephant N 10°W true

about 12°E

Noon Clarence Pk bore ^ N 22°W(mag) abt 44m = 61° 56′ 53° 5{6′}

1.0P.M. Launched boats 1.30PM. Under way to 6.15 P.M. N 21°W 11 mls

& dist

Course ^ made good approx. N 45°W 7 miles

Above DR Long & Co. being
from a bearg : of land should xxx
12′½W of prev: Obs Positions (xxx)

Monday 10th April

PM 9th

DR N45°W 7m	4.9	4.9	
to 2P.M. 10th			61° 46′ 5[4° 57′]
DR N60°W 10m	5.0	8.7	N71°W[30 mls]
Estd Curt W 15m		15	
	9.9	28.6	

Tuesday 11th April

Estd Curt W 30 m C. Melville 62° 2′ 57° 33′ — 61° 46′ [56° 0′]

16′S 95

446 — C. Melville S7

Wednesday 12th April

slow 9 1				
24.18 19. O	9° 54 AM	S 37° W 10m 8′S. 13′W		62° 15′ {53° 7′}
- 51	62 7	33.006		
24 17 28	98 40	.498		
20 45 11	170 41	90963		
3 32 7	85 20½	98583		
53° 2′	75 26½	23050		

24.17 26.	9° 50 AM	S 37° W 8m	6 .35 .29 O	10° 46 PM	S 40° W 5m
- 51	62.10	330.78	- 47	62.19	33.294
24 16 35	98 .40	4.99	6 34 .42	98 .46	.510
20 44 21	170 .40	910.40	3 2 2	170 .51	85.164
3 32 14	85 .20	987 83	3 .32.40	85 .55½	98.526
53° 3′ 30″	76.30	234 00	53°10′	75 . 9½	17494

Thursday 13th Jan

S 30 W 5	4.3	2.5	61° 43′	54°36′
N 33° W 10	8.4	5.4	61 11	54 50
N 45° W 40 to 8PM	28.3	28.3	32	14
	32.4	36.2		64 S

Annotated Log

Friday, 7th April

Noon Position 62°08′S, 54°22′ W

Bearing and Distance

Noon Position	62 °	8 ′	S	54 °	22 ′	W	D.Lat.	D.Lon.	Dep.	Bearing	Distance
Clarence Peak	61	12		53	40		56.0	42.0	19.9	20 °	59 miles
Bridgeman Island	62	11		56	25		-3.0	-123.0	-57.4	267 °	58 miles

Saturday, 8th April

Noon Position

DR N82°E 16 miles from 62°8′ S, 54°22′ W: 62°06′ S, 53°48′ W

D. Lat. 2.2 Dep. 15.8 = D. Lon. 33.9

Bearing and Distance

Noon Position	62 °	6 ′	S	53 °	48 ′	S	D.Lat.	D.Lon.	Dep.	Bearing	Distance
Clarence Peak	61 °	12 ′		53 °	40 ′		54.0	8.0	3.8	4 °	54 miles

Sunday, 9th April

Noon Position

DR N21°W 11 miles from 62°6′ S, 53°48′ W: 61°56′ S, 53° 56′ W

D. Lat. 10.3 Dep. 3.9 = D. Lon. 8.4

Longitude

Clarence Peak		61 ° 12 ′ S	53 ° 40 ′ W	
Latitude		61 ° 56 ′ S		
D. Lat. 44.0	Dep. 7.9 = D. Lon. 16.3			
Longitude			53 ° 56 ′ W	

Monday, 10th April

Noon Position

Wednesday, 12th April

Noon Position 62°15′ S, 53° 7′ W

Time Sight

Mean time at Greenwich	24 h	18 m	19 s	Sun's true altitude	9 °	54.0 ′	AM	
Equation of Time	-		51	Latitude	62	7.0	sec.	0.33006
Apparent time at Greenwich	24	17	28	Polar distance	98	40.0	cosec.	0.00499
				Sum	170	41.0		
Apparent time at ship	20	45	11	Half-sum	85	20.5	cos.	8.90963
Longitude in time	3	32	17 W	Remainder	75	26.5	sin.	9.98583
Longitude	53 °	4 ′	15 ″				hav.	9.23051

Time Sight

Mean time at Greenwich	24 h	17 m	26 s	Sun's true altitude	9 °	50 ′	AM	
Equation of Time	-		51	Latitude	62	10	sec.	0.33078
Apparent time at Greenwich	24	16	35	Polar distance	98	40	cosec.	0.00499
				Sum	170	40		
Apparent time at ship	20	44	48	Half-sum	85	20	cos.	8.91040
Longitude in time	3	31	47 W	Remainder	75	30	sin.	9.98594
Longitude	52 °	56 ′	45 ″				hav.	9.23211

Run to Noon S 37° W 8 miles

Time Sight

Mean time at Greenwich	6 h	35 m	29 s	Sun's true altitude	10 °	46.0 ′	PM	
Equation of Time	-		47	Latitude	62	19.0	sec.	0.33294
Apparent time at Greenwich	6	34	42	Polar distance	98	46.0	cosec.	0.00510
				Sum	171	51.0		
Apparent time at ship	3	2	2	Half-sum	85	55.5	cos.	8.85164
Longitude in time	3	32	40 W	Remainder	75	9.5	sin.	9.98526
Longitude	53 °	10 ′	0 ″				hav.	9.17494

Run from Noon S 40° W 5 miles

<u>Thursday, 13th April</u>

8PM Position

DR N48°W 49 miles from 62°15′ S, 53°19′ W: <u>61°43′ S, 54°36′ W</u>

D. Lat. 32.4 Dep. 36.2 = D. Lon. 77.1

				D.Lat.	Dep.	D.Lon.
S	30° W	5	miles	4.3	2.5	
N	33° W	10	miles	8.4	5.4	
N	45° W	40	miles to 8PM	28.3	28.3	
				32.4	36.2	77.1

Appendix B

Excerpts of tables from the *Nautical Almanac* (1915) consulted in the reduction of the occultation of 42 Libræ on 24 June 1915. The times listed are astronomical time with 0h occurring at noon on the date in question

II. **JUNE, 1915.** 63

AT MEAN NOON.							
Date.		THE SUN'S *Apparent* Right Ascension.	THE SUN'S *Apparent* Declination.	THE SUN'S Semi-diameter.*	Equation of Time, *to be subtracted from* / *added to Apparent Time.*	Sidereal Time.
		h m s	° ′ ″	′ ″	m s	h m s
Tues.	1	4 33 2·57	N. 21 56 51·9	15 47·75	2 30·55	4 35 33·12
Wed.	2	4 37 7·93	22 5 8·9	15 47·61	2 21·75	4 39 29·68
Thur.	3	4 41 13·70	22 13 2·8	15 47·47	2 12·54	4 43 26·24
Tues.	22	5 59 54·90	23 27 7·1	15 45·67	1 34·06	5 58 20·85
Wed.	23	6 4 4·39	23 26 55·2	15 45·63	1 46·99	6 2 17·40
Thur.	24	6 8 13·81	23 26 18·5	15 45·59	1 59·85	6 6 13·96

* The Semidiameter for *Apparent* Noon may be assumed the same as that for *Mean* Noon.

64 **JUNE, 1915.** III.

MEAN TIME.

Day.	THE SUN'S *Apparent* Longitude. *Noon.*	THE SUN'S *Apparent* Latitude. *Noon.*	Logarithm of the Radius Vector of the Earth. *Noon.*	Transit of the First Point of Aries.	THE MOON'S Semidiameter. *Noon.*	THE MOON'S Semidiameter. *Midnight.*	THE MOON'S Horizontal Parallax. *Noon.*	THE MOON'S Horizontal Parallax. *Midnight.*
	° ′ ″			h m s	′ ″	′ ″	′ ″	′ ″
1	69 54 26·3	S. 0·07	0·0061103	19 21 16·11	16 12·10	16 4·39	59 21·52	58 53·31
2	70 51 54·2	N. 0·05	·0061767	19 17 20·20	15 56·52	15 48·67	58 24·48	57 55·67
3	71 49 21·4	0·19	·0062415	19 13 24·29	15 40·95	15 33·50	57 27·40	57 0·12
22	89 58 49·9	0·41	0·0070631	17 58 41·95	16 13·79	16 19·81	59 27·73	59 49·83
23	90 56 3·2	0·45	·0070844	17 54 46·04	16 25·35	16 30·23	60 10·12	60 28·01
24	91 53 15·8	0·45	·0071041	17 50 50·12	16 34·30	16 37·41	60 42·90	60 54·27

X. **JUNE, 1915.** 71

MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.	Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Tuesday 22.					**Thursday 24.**			
0	13 23 57·23	22·366	S. 13 53 20·0	144·78	0	15 20 33·10	26·292	S. 23 41 54·2	92·32
1	13 26 11·64	22·438	14 7 46·9	144·17	1	15 23 11·09	26·372	23 51 3·3	90·70
2	13 28 26·49	22·512	14 22 10·0	143·53	2	15 25 49·56	26·451	24 0 2·6	89·07
3	13 30 41·78	22·585	14 36 29·3	142·89	3	15 28 28·50	26·528	24 8 52·1	87·42
4	13 32 57·51	22·659	14 50 44·7	142·23	4	15 31 7·90	26·604	24 17 31·6	85·73
5	13 35 13·69	22·734	15 4 56·0	141·53	5	15 33 47·75	26·680	24 26 0·9	84·03
6	13 37 30·32	22·810	15 19 3·1	140·83	6	15 36 28·06	26·755	24 34 20·0	82·33

452 **MEAN PLACES OF OCCULTATION STARS, &c., 1915.**

Star's Name.	Mag.	Assumed Right Ascension, 1915·0	P.M. in R.A.	Assumed Declination, 1915·0	P.M. in Dec.
		h m s	s	° ′ ″	″
B.A.C. 4225	6·3	12 27 16·45	− ·002	− 4 35 1·5	+ ·04
q Virginis	5·4	12 29 23·45	− ·006	− 8 58 59·4	·00
χ Virginis	4·8	12 34 51·45	− ·006	− 7 31 40·7	− ·03
B.D. − 10° 3570	6·c	12 49 53·19	− ·006	− 11 11 16·5	− ·04
ψ Virginis	4·9	12 49 55·84	− ·002	− 9 4 39·2	− ·03
42 Libræ	5·1	15 35 15·18	− ·002	− 23 32 33·3	− ·03
b Scorpii	4·8	15 45 51·78	− ·002	− 25 29 37·7	− ·04
A Scorpii	4·7	15 48 30·32	− ·002	− 25 4 26·4	− ·02
B.A.C. 5253	5·4	15 48 49·03	− ·002	− 24 16 50·6	− ·04
3 Scorpii	5·9	15 49 33·08	− ·003	− 24 59 32·9	− ·03

478 **ELEMENTS OF OCCULTATIONS, 1915.**

Star's Name.	Magnitude.	Reduction to Date. Δ α	Reduction to Date. Δ δ	Greenwich Mean Time of Conjunction in R.A.	Greenwich Hour-Angle. West + East −	$q_{\circ}$	p'	q'	Limits of Latitude.
		s	″	June d h m s	h m				° °
39 Cancri	6·5	+1·35	+ 0·9	16 1 12 15	− 1 48	−0·5408	·5276	−·1861	14 N. 63 S.
40 Cancri	6·5	+1·35	+ 0·9	1 14 46	− 1 46	−0·5091	·5276	−·1861	16 N. 62 S.
B.A.C. 5023	5·7	+3·36	−19·3	20 32 31	+11 28	−1·0483	·5914	−·1607	29 S. 90 S.
B.A.C. 5111	6·3	+3·49	−18·8	24 2 53 13	− 6 28	+0·0743	·5972	−·1437	31 N. 39 S.
42 Libræ	5·1	+3·52	−18·3	5 34 2	− 3 54	−0·9521	·5996	−·1362	25 S. 90 S.
b Scorpii	4·8	+3·62	−18·0	9 31 1	− 0 7	+0·4554	·6029	−·1248	51 N. 18 S.